TDA Data Management Planning: Construction of Maximal Daily Tracking Schedules

C. A. Greenhall
Consultant to the TDA Planning Office

An algorithm is described that computes time-per-day available to each of several spacecraft that are tracked simultaneously by a DSN subnet.

I. Introduction

The algorithm described herein was written as a tool for use by the DSN in making long-range load forecasts. The input data are a set of view periods of several spacecraft as seen by each of the three stations of a subnet during a particular 24-hour period. These view periods can overlap in complicated ways; thus it is a substantial problem to compute how much time can be given to each spacecraft (S/C), given that all the S/C must be equally well covered. More precisely, let us number the S/C from 1 to n. If S/C number s is tracked for $c(s)$ hours by the subnet, then the figure of merit used in this article for the tracking schedule is

$$m = \min_{s} c(s)$$

We seek a tracking schedule that makes m as large as possible.

A variant of this problem requires that one of the missions, say $s = s_f$, be given 24-hour coverage. In this case, the minimum is taken over all $s \neq s_f$.

The problem is solved by converting it to a linear program. Unfortunately, the linear program is not quite a correct model of the problem, for in some cases the linear program is not feasible. If certain constraints of the problem are then relaxed, the linear program usually becomes feasible and provides useful estimates. In addition, there is a separate computation, valid for all cases, that quickly gives an upper bound on S/C coverage time.

The algorithm has been imbedded in a conversational, structured MBASIC program, called DATRAMAX (DAily TRAcking MAXimizer). This form is convenient for evaluating the usefulness of the algorithm.

II. Problem Statement

A subnet of Deep Space Stations at Goldstone, Australia, and Spain is to track n spacecraft where $2 \leq n \leq 6$. The view period of each S/C at each station is given and is assumed to be constant from day to day in this model. Also given are the following nonnegative parameters: ϵ, the minimum elevation for tracking; τ, the minimum time for a station to transfer its attention from one S/C to another; and δ, the minimum duration of a tracking period. One of the S/C may be *favored*, that is, it may be required that it be tracked by one station whenever *some* station sees it above elevation ϵ.

It is assumed that the time for a S/C to go from the horizon to elevation ϵ is approximately independent of the station and S/C locations; thus, ϵ will be given in units of time and assumed to be constant. In fact, it is sufficient simply to shorten the ends of each view period by ϵ; we will speak then of *shortened* view periods.

A *feasible tracking schedule* is defined as follows. For each station and S/C, the station either does not track the S/C at all, or tracks it during a single subinterval of the shortened view period. The schedule is periodic with period 24 hours. At any time, no two stations track the same S/C and, of course, a station cannot track more than one S/C. The tracking periods satisfy the constraints implied by the numbers τ and δ. One of the S/C may be favored as mentioned above.

For any such schedule, set $c(s)$, $s = 1$ to n, equal to the total time devoted to S/C s by all three stations. The problem is to find the greatest minimum coverage

$$\max\left(\min_{s \neq s_f} c(s)\right)$$

where the maximum is taken over the collection of feasible tracking schedules, and s_f is the index of the favored S/C. (If none are favored, then s_f is set equal to 0.)

III. Solution Technique

Let us label the Stations 1, 2, 3 (Goldstone, Australia, Spain) and the S/C from 1 to n.

This algorithm solves the problem for most practical cases. The exceptions occur because of additional restrictions that are placed on the class of feasible schedules. First, we assume that each shortened view period contains a nonempty tracking period (even if that tracking period reduces to a single point!) For example, if $\delta = 3$, then every view period must contain a tracking period of at least 3 hours—no tracking period can be omitted. For certain combinations of view periods, ϵ, τ, δ, and s_f, this assumption forces the constraints to become incompatible. For example, suppose that there are two S/C at the same place in the sky, and that their view periods are 2000–0900 at Goldstone, 0400–1300 at Australia, and 1200–0100 at Spain (Fig. 1). (This is an approximation of a set of real view periods.) Assume for this demonstration that $\epsilon = 0$, $\tau = 0$, $\delta = 3$, and that S/C 1 is favored. Then S/C 1 *must* be tracked by Goldstone at least between 0100 and 0400, by Australia at least between 0900 and 1200, and by Spain at least between 1300 and 2000, and during these periods, S/C 2 cannot be tracked. Therefore, S/C 2 can be tracked only during the periods 0400–0900, 1200–1300, and 2000–0100. But it is impossible to encase three disjoint tracking intervals of at least 3 hours each within the union of these three periods.

For most view period data, the constraints are compatible if τ and δ are set to 0.

The second restriction is less serious. For each station, the n tracking periods are disjoint subintervals of the cyclic 24-hour day; we assume that they occur in the same cyclic order as the midpoints of the Goldstone view periods. Also, the three tracking periods of each S/C are assumed to follow each other in the east-west cyclic order 1, 2, 3, 1. We feel that these assumptions are not unduly restrictive, for the configuration of view periods does not change much from station to station, and this choice seems to allow the most room for placing the tracking periods.

These additional restrictions allow the problem to be reduced to a linear program whose variables give the left and right end points of the tracking periods. Let the shortened view period of station a and S/C s have rise and set times $\rho(a, s)$ and $\sigma(a, s)$. These times can be shifted by multiples of 24 so that $0 \leq \rho(a, s) < 24$, $\rho(a, s) < \sigma(a, s) < \rho(a, s) + 24$. Let the tracking period for the combination (a, s) be

$$(\rho(a, s) + z(a, s), \sigma(a, s) - w(a, s))$$

where $z(a, s)$ and $w(a, s)$ are the nonnegative linear program variables.

Following are the linear constraints and object function:

(1) *View constraints:* these state that the tracking periods have length $\geq \delta$:

$$z(a, s) + w(a, s) \leq \sigma(a, s) - \rho(a, s) - \delta$$

for all a, s.

(2) *Possibility constraints:* these state that the tracking periods for a station follow each other in the cyclic midpoint order mentioned above. Let $m(s)$, $s = 1$

to n, be the midpoint of the Goldstone view period $(\rho(1,s), \sigma(1,s))$. Let $s_1, \cdots, s_n$ be a permutation of $1, 2, \cdots. n$ such that $m(s_1) \leq m(s_2) \leq \cdots \leq m(s_n)$. Let $s_{n+1} = s_1$,

$$\begin{aligned} \rho'(1, s_{j+1}) &= \rho(1, s_{j+1}), && \text{if } j < n \\ &= \rho(1, s_1) + 24, && \text{if } j = n \end{aligned}$$

For $a = 2$ and 3, determine $\rho'(a, s_{j+1})$ such that

$$\rho'(a, s_{j+1}) \equiv \rho(a, s_{j+1}) \qquad (\text{mod } 24),$$
$$-12 < \sigma(a, s_j) - \rho'(a, s_{j+1}) - \sigma(1, s_j) + \rho'(1, s_{j+1}) < 12$$

This ties the view period configurations of stations 2 and 3 to that of station 1. The constraints are

$$w(a, s_j) + z(a, s_{j+1}) \geq \sigma(a, s_j) - \rho'(a, s_{j+1}) + \tau$$

for $a = 1$ to 3, $j = 1$ to n. If the right-hand side is not positive, that constraint is omitted.

(3) *Redundancy constraints:* these express the east-west order of the tracking periods of each S/C. Let $\oplus$ be defined by

$$\begin{aligned} a \oplus 1 &= a + 1, \qquad \text{if } a = 1 \text{ or } 2 \\ 3 \oplus 1 &= 1 \end{aligned}$$

For $a = 1$ to 3, $s = 1$ to n, determine $\rho''(a \oplus 1, s)$ such that

$$\rho''(a \oplus 1, s) \equiv \rho(a \oplus 1, s) \qquad (\text{mod } 24),$$
$$0 < \rho''(a \oplus 1, s) - \rho(a, s) < 24$$

The constraints are

$$w(a, s) + z(a \oplus 1, s) \geq \sigma(a, s) - \rho''(a \oplus 1, s)$$

for $a = 1$ to 3, $s \neq s_f$. (Omit any constraint with nonpositive right-hand side.) If $s_f \neq 0$, then the constraints for $s = s_f$ are

$$w(a, s_f) + z(a \oplus 1, s_f) = \max(0, \sigma(a, s_f) - \rho''(a \oplus 1, s_f))$$

for $a = 1$ to 3. (If the right-hand side is zero, we could eliminate $w(a, s_f)$ and $z(a \oplus 1, s_f)$ from the linear program, but the computer program would become more complex.)

(4) *Object constraints and object function:* The object function to be maximized is an auxiliary variable μ, which satisfies the constraints

$$c(s) \geq \mu, \qquad s \neq s_f$$

where

$$c(s) = \sum_{a=1}^{3} (\sigma(a, s) - \rho(a, s) - z(a, s) - w(a, s))$$

the tracking time devoted to s by all three stations.

This linear program is solved by a general simplex routine, which, of course, reports the infeasibility of the constraints, if such is the case. Once the linear program is solved, it is easy to make a further improvement not called for by the problem statement. Let $m = \max \mu$, obtained by the previous linear program. Consider another linear program with the same variables, the same constraints (1) through (3), but with (4) replaced by

(4)′ *Object constraints:*

$$c(s) \geq m + \mu, \qquad s \neq s_f$$

Object function:

$$\sum_{s \neq s_f} c(s)$$

The final solution of the first program, with μ set to 0, is given to the second program, which maximizes total tracking time while keeping each $c(s)$ not less than the best minimum coverage per S/C. The initial tableau of the second program differs very little from the final tableau of the first program, and the optimization of the second program requires few simplex iterations.

Because the algorithm sometimes causes incompatible constraints, a backup computation is provided for all cases. This gives (1) an upper bound on total tracking time, (2) time available to the favored S/C if $s_f \neq 0$, and (3) an upper bound on tracking time per unfavored S/C. The set of endpoints of the shortened view periods divides the cyclic 24-hour day into subintervals called *atoms*. For the kth atom, there is an incidence table $E_k(a, s)$, where $E_k(a, s) = 1$ if atom k is contained in the shortened view period of station a and S/C s, and $E_k(a, s) = 0$ otherwise. A *diagonal* of a 0–1 matrix (such as E_k) is defined as a subset D of the set of "1's" of the matrix such that D has no two 1's in the same row or column. (This generalizes the ordinary notion of the diagonals of a square matrix.) Let a feasible tracking schedule be constructed upon the given view periods. At any instant of time t during the k^{th} atom, the set of index pairs (a,s) such that station a is tracking S/C s must be a diagonal of E_k. Hence, the number of S/C being tracked at time t is bounded above by the size d_k of a *largest* diagonal of E_k. Therefore, if ℓ_k is the length of atom k, the total tracking time of any feasible schedule cannot exceed $B = \Sigma d_k \ell_k$.

Since E_k is a 3 by n matrix, the number d_k can equal only 0,1,2, or 3, and a simple search suffices to discover its value. If there is a favored S/C $s_f \neq 0$, and if E_k has a 1 in column s_f, then only those diagonals of E_k that have a 1 in this column can be considered. It can be proved, however, that the maximum size of the diagonals in this restricted set again equals d_k.

The time T_f available to S/C s_f is simply $\Sigma \ell_k$, where the sum is over all k such that E_k has a 1 in column s_f. Then $B - T_f$ divided by the number of unfavored S/C is a bound on the tracking time for each of these S/C.

IV. Sample Inputs and Outputs

Suppose that there are 2 S/C with the following view periods:

	Goldstone		Australia		Spain	
	Rise	Set	Rise	Set	Rise	Set
S/C 1	0430	1748	1340	2136	2050	1033
S/C 2	2225	1046	0615	1540	1450	0327

Let $\epsilon = 0.5$ h, $\tau = 1$ h, $\delta = 3$ h, $s_f = 0$.

The "atoms" calculation yields a bound 46.02 h on total tracking and hence, a bound 23.01 h on tracking time per S/C. The linear programs compute the following tracking schedule and coverages:

	Goldstone	Australia	Spain
S/C 1	1003–1610	1610–2106	2235–1003
S/C 2	2255–0903	0903–1510	1520–2135

S/C 1:	22.51 h
S/C 2:	22.51 h
Total	45.02 h

Now let $s_f = 1$. The "atoms" calculation shows 23.77 h available to the favored S/C 1, and hence, a bound of 22.25 h on tracking time for S/C 2. The tracking schedule and coverages are now

	Goldstone	Australia	Spain
S/C 1	1003–1620	1610–2106	2120–1003
S/C 2	2255–0903	0903–1510	1520–2020

S/C 1:	23.77 h
S/C 2:	21.25 h
Total	45.02 h

V. Conclusions

This algorithm has two main defects caused by forcing the problem into the framework of one or two fixed linear programs. First, the linear program can be infeasible. Second, the schedules produced cannot be described as "optimal." More criteria are needed. For example, having maximized the smallest $c(s)$ (coverage of S/C s), one could then maximize the next smallest $c(s)$, and so on. Remedy of these defects may require programming with mixed real and integer variables.

In spite of these defects, the algorithm will still yield realistic estimates of time available to future missions.

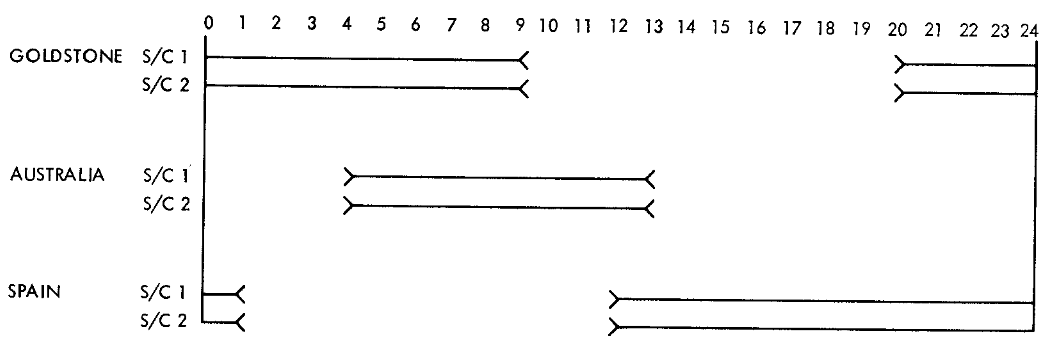

Fig. 1. An example of incompatible constraints